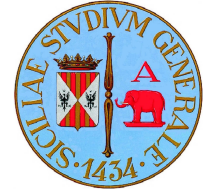




Ph.D. in Economics, Management & Statistics
University of Catania & University of Messina



Microeconomics

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Dr. D. Maimone Ansaldo Patti

Student's Name: _____

Signature: _____

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Instructions: The exam paper consists of two sections. Answer **ONE** question from each section. If you answer more questions from the same section, only the first answer will be marked. You can use standard calculators. Be sure that your mobile phone is switched off and kept in your bag. All the bags should be stored in the place indicated by the invigilator. When you sit in the designed place, be sure that you do not have notes and other unauthorized material. If you have such a material, you should store it in your bag. If unauthorized material will be found in your possession after the beginning of the exam, you will be asked to leave quietly the room, failing the exam. You have 2.5 hours to complete your exam. Before starting, you have 10 minutes to read the exam paper. If you have any question, raise your hand and ask the invigilator. You **must use** the papers provided to write your exam. If you need additional papers, ask the invigilator. When handling your exam, remember to write your name both on the exam paper and on those where you wrote your answers. You pass the exam if your average mark is **50 or above**.

Section A

Question 1: 100 marks

The preference structure of a consumer is represented by the following Cobb–Douglas utility function:

$$U = U(x_1, x_2) = x_1^\alpha x_2^{1-\alpha}$$

where $0 < \alpha < 1$.

- (a) [10 Marks] Are preferences represented by the Cobb–Douglas utility function homothetic? Explain.
- (b) [20 Marks] By solving the utility maximization problem, check that the Marshallian demand functions are given by $\mathbf{x} = [x_1^*(\mathbf{p}, m), x_2^*(\mathbf{p}, m)] = \left[\frac{am}{p_1}, \frac{(1-\alpha)m}{p_2} \right]$
- (c) [10 Marks] Find the indirect utility function and check that the indirect utility function is homogeneous (of which degree?) in prices and income; what is the economic interpretation of this homogeneity property?
- (d) [10 Marks] Check the validity of Roy's identity.
- (e) [30 Marks] Find the Hicksian demand functions and show the symmetry of the cross price effects.
- (f) [20 Marks] Check that the expenditure function is given by $e(\mathbf{p}, m) = K p_1^\alpha p_2^{(1-\alpha)}$ (where K is some constant depending on the parameter α). Check then if the expenditure function is increasing and homogeneous of degree 1 in prices and show that the Shepard lemma holds.

Question 2: 100 marks

Let us consider the following production function:

$$q = f(x, z) = [x^a + z^a]^{\frac{1}{2a}}$$

where $a > 0$.

- (a) [10 Marks] Determine the returns to scale of such a technology.
- (b) [10 Marks] Does this production function has a homothetic isoquant curve map?
- (c) [20 Marks] Find the TRS and the elasticity of substitution between x and z ; what does the elasticity of substitution measure? Explain.

- (d) [30 Marks] From now on, assume $a = 0.5$ and let $\mathbf{w} = [w_x, w_z]$ be the input price vector. Find the corresponding minimum cost function (assuming that the conditions for an interior solution are met).
- (e) [20 Marks] Show that the usual properties hold for this minimum cost function (increasing in q ; increasing and concave in any component of the price vector \mathbf{w} , homogeneous of degree 1 in \mathbf{w} , the Shephard lemma).
- (f) [10 Marks] Find the average and the marginal cost functions.

Section B

Question 3: 100 marks

Consider the following game:

		P2		
		L	C	R
P1	T	2, 0	1, 4	4, 2
	M	0, 4	1, 0	2, 3
	B	1, 3	4, 2	3, 0

- (a) [20 Marks] Explain the Nash equilibrium solution concept. Is this solution concept always Pareto efficient? Discuss.
- (b) [30 Marks] Check whether there exist a Nash equilibrium in pure strategy. Explain carefully the steps you take. (**HINT:** Do not write down simply your solution, but explain how you did reach your answer)
- (c) [50 Marks] Check whether $(\sigma_1^*, \sigma_2^*) = \left\{ \left(\frac{10}{25}, \frac{11}{25}, \frac{4}{25} \right); (0, 1, 0) \right\}$ could be a possible solution of the game in mixed strategy.

Question 4: 100 marks

Suppose that three firms 1, 2 and 3 operate in a market. They choose the quantity, $q_i \in [0, \infty) \forall i = 1, 2$ and 3, that maximize their profit functions. The price is given by:

$$p(q_i, q_j, q_k) = a - Q$$

where $Q = q_i + q_k + q_j$. The profit function takes the following form:

$$\pi_i = p(q_i, q_j, q_k) q_i - cq_i \quad \forall i = 1, 2, 3$$

There exists perfect information among firms.

- (a) [5 Marks] Write down the maximization problem for each firm.
- (b) [25 Marks] Calculate the quantities that each firm chooses in equilibrium.
- (c) [15 Marks] Calculate the profits enjoyed by each firm.
- (d) [15 Marks] Now suppose that firm 1 chooses its quantity first, while firms 2 and 3 choose their quantity simultaneously after having observed the choice made by firm 1. The game is, therefore, sequential. Discuss the solution concept that you apply in this case.
- (e) [40 Marks] Calculate the quantities that each firm chooses in equilibrium under the scenario in d). Contrast your solution with the one that you derived in point b).

END